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THESIS

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by

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Interaction of Two Sound Beams in Water

by

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ABSTRACT

Modern sonars are large in size. This is a result of several factors connected with operation at low frequencies. Non-linear interaction of two high-frequency sound beams in water may provide a source of low-frequency sound without the use of large transducers. The theory of interaction of two plane waves is reviewed, and conversion efficiency is studied. Early experimental work is also reviewed. To provide new experimental data, primary sources of 385 kHz and 435 kHz are mounted side-by-side in an anechoic tank. The sound field between the parallel beams is explored. A difference-frequency signal is detected, but its amplitude is found to be greater than the theoretical value. Non-linearities in the receiving hydrophone are suspected as contributing to the 50-kHz signal level. Various techniques for eliminating psuedo-sound are discussed, and recommendations for further study are presented.

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I. INTRODUCTION

The history of surface-ship sonars has been characterized by ever-increasing size. This is a direct result of the lowering of operating frequency to take advantage of reduced attentuation in the medium. As signal frequency is lowered and wave length is increased, the transducer elements become large in order to maintain dimensions comparable to the wavelength. The lowering of sonar frequencies has also aggravated the problem of cavitation limiting. Since the cavitation threshold is lowered as frequency is lowered, the power that can be transmitted through a given area is diminished. Figure 1 shows that for 20 kHz, this threshold is only about 2 W/cm^2. An increase in pulse length also lowers the threshold as shown in Fig. 2. Since mechanical steering of large transducers is impractical, electrical steering of large arrays to form beams has to be employed. This further aggravates the cavitation problem by producing large pressure peaks relative to the average distribution over the array.

If attenuation were not a limiting factor, sonar operation at frequencies of a few hundred kilohertz would be attractive. Shorter wavelengths would permit use of smaller elements, and as seen in Fig. 1, cavitation thresholds might be raised by a factor of ten or more. The resulting smaller transducer might be mechanically steerable, eliminating the pressure peaking of electrical steering. A reduction in transducer size would produce obvious weight and structural advantages in a surface

ship and would probably lower costs as well. In some applications, such as in active sonobuoys or acoustic torpedoes, small transducers are mandatory. An increase of operating frequency there could lead to smaller overall size, or transformation of space to power systems use, etc.

The non-linear interaction of two high-frequency sound beams in water may be a useable source of low-frequency energy. The advantages of high-frequency operation as well as those of low-frequency operation could be exploited simultaneously. In this process, stresses are established because of the different instantaneous particle velocities associated with the two waves. These stresses act to produce an acoustic source at the difference frequency. Hence, small transducers might be used to produce two high-frequency signals which would act to produce a low-frequency signal. The latter signal would propagate without suffering the large attenuation which limits high-frequency operation.

Although a considerable amount of theoretical work has been done in this area, the experimental work which has been reported is limited in scope. Most of the work which has been done in water has been at frequencies above one megahertz. While beamwidth has been studied closely, little data exist on the absolute amplitude of the difference-frequency signals. The results of some work are clouded by psuedo-sound, which is developed due to non-linear summing at the hydrophone. The typical response of a non-linear device will contain functions of the second power. If the hydrophone is non-linear, the radiation pressures

of two incident waves may produce a signal with frequencies including the difference frequency. Separation of psuedo-sound and the true difference frequency signal has seldom been accomplished by direct experimental means.

Most proposals for use of the low-frequency signal produced by high-frequency sources have concerned narrow-beamwidth sonar, frequency-scanning sonars, or wide-bandwidth applications. Parametric amplifiers have also been considered. References 1-4 suggest some of the interesting possibilities.

The purpose of this work was to produce and measure a desired difference-frequency signal while using primary sources of a few hundred kilohertz. In pursuit of this goal, it was desired to determine the characteristics of psuedo-sound and to explore methods for its elimination in the problem.

II. THEORY

The generation of sound by the non-linear interaction of two sound waves is best described by starting with the theory of sound generated aerodynamically. Lighthill, in Ref. 5, begins with the fundamental equations governing the propagation of sound. For a uniform non-viscous medium, without sources of matter or external forces, the linearized equations are: the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u_i}{\partial x_i} = 0 \; ; \tag{1}$$

the equation of conservation of momentum,

$$\rho_0 \frac{\partial u_i}{\partial t} + \frac{\partial p}{\partial x_i} = 0 \; ; \tag{2}$$

and the equation of state,

$$p = \rho \, c_0^2 \; ; \tag{3}$$

where ρ, u, c_0 and p are the acoustic parameters. The resulting wave equation is

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = 0 \, . \tag{4}$$

If externally applied fluctuating stresses are present, then a stress term must be added to Eq. (2),

$$\rho_0 \frac{\partial u_i}{2t} + \frac{\partial p}{\partial x_i} = - \frac{\partial T_{ij}}{\partial x_j} \, . \tag{5}$$

Equation (4) then becomes

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \, . \tag{6}$$

For the case of a fluctuating fluid flow within a larger body of fluid at rest, Lighthill shows that the stress term may be expressed as

$$T_{ij} = \rho u_i u_j + p_{ij} - c_o^2 \rho \delta_{ij} . \qquad (7)$$

Here, $\rho u_i u_j$ is the momentum flux tensor (Reynolds stress tensor) and p_{ij} is the compressive stress tensor. The last term represents the stresses of a simple hydrostatic pressure field. Viscous stresses and heat conduction are neglected. The fluctuating Reynolds stresses, corresponding to the variation in momentum flux across the fluid boundaries, and the fluctuating applied pressures serve as sources of sound. Kinetic energy is converted to acoustic energy. Lighthill shows that T_{ij} is the strength per unit volume of a relatively weak quadrupole field.

Westervelt, in Refs. 6 and 7, applies Lighthill's work to the subject of two sound waves of different frequency, where u_i and u_j are taken as the instantaneous particle velocities of the two waves. Using Eq. (7), Westervelt rewrites Eq. (6) as

$$\Box^2 p = \frac{\partial^2}{\partial t^2} \left(\rho - \frac{p}{c_o^2}\right) - \frac{\partial^2}{\partial x_i \partial x_j} (\rho u_i u_j) . \qquad (8)$$

He then shows that pressure p_s in the difference-frequency wave can be expressed as

$$\Box^2 p_s = - \frac{1}{2c_o^6} \left(\frac{d^2p}{d\rho^2} \right)_{\rho=\rho_o} \frac{\partial^2}{\partial t^2} (p_i^2) - \rho_o \nabla^2 u_i^2 . \qquad (9)$$

Here, p_s is given as a function of the primary-wave variables which are subscripted i. The substitutions

$$u_i = \frac{p_i}{\rho_o c_o} \quad \text{and} \quad \nabla^2 p_i^2 = \Box^2 p_i^2 + \frac{1}{c_o^2} \frac{\partial^2}{\partial t^2} (p_i^2) \qquad (10)$$

lead, finally, to the useful expression

$$\nabla^2 p_s = - \rho_o \frac{\partial q}{\partial t} \quad , \tag{11}$$

where the source density function is

$$q = \frac{1}{\rho_o^2 c_o^4} \left[1 + \frac{1}{2} \frac{\rho_o}{c_o^2} \left(\frac{d^2 p}{d\rho^2} \right)_{\rho = \rho_o} \right] \frac{\partial}{\partial t} (p_i^2) \quad . \tag{12}$$

For a given temperature, the expression in brackets is a constant. In water, its approximate value is three [Ref. 2]. For convenience, it is taken as unity in what follows.

If the primary wave is now taken as the sum of two colinear plane waves travelling to the right, acoustic pressure at a distance x may be taken as

$$p_i = p_1 + p_2 = P_1 e^{-\alpha_1 x} \cos(\omega_1 t - k_1 x) + P_2 e^{-\alpha_2 x} \cos(\omega_2 t - k_2 x) \; . \tag{13}$$

The variables are the usual ones. Squaring,

$$\begin{aligned} p_i^2 = P_1^2 e^{-2\alpha_1 x} \cos^2(\omega_1 t - k_1 x) + P_2^2 e^{-2\alpha_2 x} \cos^2(\omega_2 t - k_2 x) + \\ P_1 P_2 e^{-(\alpha_1 + \alpha_2)x} \{\cos[(\omega_1 + \omega_2)t - (k_1 + k_2)x] + \\ \cos[(\omega_1 - \omega_2)t - (k_1 - k_2)x]\} \; . \end{aligned} \tag{14}$$

The first two terms lead to frequencies which are twice the primary frequencies. The last term contains both sum and difference frequencies. Following the development by Berktay [Ref. 2], the difference-frequency term is expressed as

$$P_1 P_2 e^{-(\alpha_1 + \alpha_2)x} \cos(\Omega t - K x) \quad , \tag{15}$$

where $\Omega = \omega_1 - \omega_2$ and $K = k_1 - k_2$. Then, from Eq. (12), the source density function is

$$q = \frac{-\Omega}{\rho_o^2 c_o^4} P_1 P_2 e^{-(\alpha_1 + \alpha_2)x} \sin(\Omega t - Kx) \quad . \tag{16}$$

An elemental volume will contain a source of strength $q(x,t)\,\delta x \delta y \delta z$, and the pressure at a distance r from the secondary source will be

$$\delta p_s(r,t) = -\frac{\rho_o}{4\pi r} e^{-\alpha r} \delta x \delta y \delta z \frac{\partial}{\partial t} \{ q(t - \frac{r}{c_o}) \} , \tag{17}$$

where α is the attenuation coefficient at the difference frequency. The total field at a point can be obtained by integrating the pressure field due to all such sources.

Berktay treats the sources as filling thin wafers along the axis of the two primary beams. Each wafer contains sources of identical amplitude and phase. It is assumed that the difference-frequency waves propagate linearly and that they add in phase to the right.

For the geometry of Fig. 3 and for $R >> x$, Berktay integrates Eq. (17) and finds the resulting pressure at the difference frequency:

$$p_s(R,\theta) = \frac{P_1 P_2 \Omega^2 S}{4\pi \rho_o c_o^4 R} e^{-\alpha R} \frac{\cos(\Omega t - KR - \beta)}{\{ A^2 + [2K \sin^2 \frac{\theta}{2}]^2 \}^{\frac{1}{2}}} \quad . \tag{18}$$

Symbols not heretofore defined are:

S = cross-sectional area of the wafer;

$A \approx \alpha_1 + \alpha_2 - \alpha$;

β = arctan $A/2K \sin^2(\frac{\theta}{2})$;

θ = angle from the axis;

R = distance from primary sources.

Limiting analysis to the axis of the system, the pressure amplitude is

$$P_s(R,O) = \frac{P_1 P_2 \Omega^2 S}{4\pi \rho_o c_o^4 RA} e^{-\alpha R} \quad . \tag{19}$$

Another convenient form, which follows immediately, is

$$P_s(R,O) = \frac{\sqrt{W_1 W_2}\ \Omega^2}{2\pi c_0^3 RA} e^{-\alpha R} . \qquad (20)$$

Here, W_1 and W_2 are acoustic powers in the two primary waves.

Summarizing, if two acoustic plane waves of different frequency propagate simultaneously along coincident paths, new waves are generated due to non-linear interaction. The frequency of one of the new waves is equal to the difference frequency of the primary waves. Its pressure amplitude is proportional to the product of primary-wave pressures and to the square of the difference frequency. Because of the quadrupole nature of the source, the difference-frequency signal is inherently weak.

It should be noted that a similar treatment has been given to cylindrically spreading waves. The results are comparable and will not be covered here. The plane-wave case provides adequate background for basic experimental work.

III. CONVERSION EFFICIENCY

From Eq. (20), it can be seen that the amplitude of the difference-frequency wave at a distance R is particularly sensitive to frequency. The amplitude in the source region increases with the square of frequency, but the attenuation with distance also increases with frequency. The format used by Berktay [Ref.2] is useful for demonstrating the effect of frequency, and is followed closely in this section.

Suppose it is desired to compare the pressure amplitude of the wave produced by non-linear interaction with that of a wave produced by an omni-directional source. For the omni-directional source of frequency $f_1 - f_2$ and power W_E,

$$I_E = \frac{W_E}{4\pi R^2} = \frac{P_E^2}{2\rho_o c_o} \quad . \tag{21}$$

Considering attenuation, the pressure amplitude at a distance R will be

$$P_E = \sqrt{\frac{\rho_o c_o W_E}{2\pi R^2}}\, e^{-\alpha R} \quad . \tag{22}$$

Now, for the wave produced by non-linear interaction,

$$P(R,O) = \frac{\sqrt{W_1 W_2}\, \Omega^2 e^{-\alpha R}}{2\pi c_o^3 RA} = \frac{\pi W (f_1 - f_2)^2}{c_o^3 RA}\, e^{-\alpha R} \tag{23}$$

for $W_1 = W_2 = \frac{W}{2}$. For $W = W_E$, this gives

$$\frac{P}{P_E} = \frac{W^{\frac{1}{2}}(f_1 - f_2)^2}{A c_o^3} \left(\frac{2\pi^3}{\rho_o c_o}\right)^{\frac{1}{2}} \quad . \tag{24}$$

This result differs from Eq. (2.7) of Ref. 2 by a factor of two. It is believed that Berktay's equation suffers from a simple manipulative error.

Proceeding, if $\rho_o = 10^3 \text{kg/m}^3$ and $c_o = 1500$ m/sec,

$$\frac{P}{P_E} = 1.93 \times 10^{-12} \left[\frac{W^{\frac{1}{2}} (f_1 - f_2)^2}{A} \right] . \tag{25}$$

This differs from Berktay's Eq. (2.8) by a factor of almost twenty. Again, a simple manipulative error is suspected.

Two examples will serve to demonstrate the frequency effect.

Example 1. Let $f_1 = 435$ kHz and $f_2 = 385$ kHz so that $f_1 - f_2 =$ 50 kHz and $A \approx 2 \times 10^{-2}$ nepers/m. If $W = 100$ watts, then

$$\frac{P}{P_E} = \frac{(1.93 \times 10^{-12}) (100)^{\frac{1}{2}} (50 \times 10^3)^2}{2 \times 10^{-2}} = 2.4.$$

Example 2. Let $f_1 = 20$ kHz and $f_2 = 19$ kHz so that $f_1 - f_2 =$ 1 kHz and $A \approx 6 \times 10^{-4}$ nepers/m. If $W = 100$ watts, then

$$\frac{P}{P_E} = \frac{(1.93 \times 10^{-12}) (100)^{\frac{1}{2}} (1 \times 10^3)^2}{6 \times 10^{-4}} = 0.03.$$

Pressure amplitude in a plane wave is intrinsically greater than that in a spherically spreading wave. Hence, the comparison of interacting plane waves with an omni-directional source must be interpreted carefully. If the interaction region is short and the observation point is well outside of this region, then the comparison of the two systems is a fair one. This is the case treated by Berktay and is best applied to primary signals in the megahertz range. For frequencies of a few hundred kilohertz, however, the interaction region may extend to a hundred meters. At this distance, pressure amplitude in the plane waves will be reduced

only by attenuation in the medium and will be down by about 60%. However, pressure in the spherically spreading waves will be down by a factor of several hundred. Thus, lowering the primary frequencies extends the interaction region, and this may inject an artificiality into a comparision with an omni-directional source. Again, ensuring that the observation point is far outside the interaction region will minimize the artificial advantage.

The difficulty in projecting plane waves as far as 100 m should be recognized. Rather large sources would be required.

It is instructive to consider the results that may be obtained using ordinary piston sources with 1/x pressure distribution in the far field. This system can be compared with the plane-wave system. If, for plane-wave interaction in a region of length x, pressure in one primary wave drops from P_1 to $P_1 e^{-\alpha_1 x}$, the equivalent pressure drop in the far field of a piston will be from P_1 to $P_1 \frac{e^{-\alpha_1 x'}}{x'}$. If the final pressures are equal, then x' can be compared with x. For $\alpha_1 = 10^{-2}$ nepers/m and x = 100 m, final pressures are equal in the two situations when $x' = 2.61$ m. For a given pressure amplitude at the beginning of each interaction region, and for equal pressures at the ends of the regions, the interaction volume with the piston sources is smaller by a factor of about 38. The effective source strength will be reduced accordingly. In the case of example 1, P/P_E becomes 0.063.

A more thorough evaluation of a piston source would include the near-field pressure pattern. Peak pressures there would be greater than

pressures in the far field, and the effective interaction volume would be increased. The far-field example provides an order of magnitude for discussing the piston source.

In the practical case, then, conversion efficiency is sensitive to primary transducer design as well as to frequency. If efficiency is important, the transducer will have to be configured to produce at least quasi-planar waves.

IV. EXPERIMENTAL PROCEDURE

A. GENERAL

As noted in the introductory section, most of the experimental work in non-linear interaction of two sound waves has been performed at megahertz frequencies. At such frequencies, the primary waves are attenuated rapidly, and a natural limit is set on the length of the interaction volume. For transducers which operate as pistons, however, the use of high frequency extends the near field or Fresnel zone. The pressure field varies widely in this region, and there must be a departure from plane-wave theory.

There is disagreement as to the effect of large cross-section of the interaction volume relative to wavelength squared at the difference frequency. Lauvstad and Tjötta, in Ref. 8, state that wavelength must be large compared to the linear dimension, or else destructive interference between the quadrupoles will predominate. Berktay [Ref. 2] allows the linear dimension to exceed wavelength, producing an aperture effect. These arguments have not been reconciled. It is generally accepted that any fluctuations of amplitude or phase across the sound beams will result in a reduction of the difference-frequency signal.

As noted earlier, psuedo-sound may enter into experimental results. Since the signal produced by non-linear summing may have the same frequency as the one supposed to be produced in the medium, and since both signals may be relatively weak, a discrimination problem occurs.

Selection of a hydrophone which does not respond to either primary frequency alone will not necessarily avoid the psuedo-sound response. This non-linear reaction is considered to occur on the surface of the unit, rather than in the sensitive elements.

B. PREVIOUS WORK

Gorelik and Zverev, in Ref. 9, reported the interaction of two perpendicular sound beams in water. The frequencies of the two signals were 1.5 MHz and 4 kHz. A detector was placed at a point in the high-frequency beam which was past the interaction region. Amplitude and phase modulation of the high-frequency signal were detected, and both were shown to be functions of the low frequency. Powers of about one watt were employed. The possibility that subsidiary effects in the equipment were causing the modulation was rejected on the grounds that detector output vanished if either beam was blocked.

In Ref. 10, Ingard and Pridmore-Brown reported a similar experiment in air. The intersecting beams were signals of either 110 kHz or 130 kHz and of 10 kHz. In the first case, the sum frequency (120 kHz) was desired and in the second case, the difference frequency (also 120 kHz) was desired. The beams intersected at a point 10 cm from each transducer. Initially, an electrostatic microphone was used as a detector. It was found that the microphone produced its own sum and difference frequencies when exposed to sound from the primary beams. A barium titanate crystal was then used as a detector and a radio receiver was used as a

selective network. With this system, it was supposed that any signal detected was a true acoustic wave. The sum/difference-frequency signal was reported at various points outside the region of interaction. Its strength was about 10-dB less than expected.

Mikhailov used 1.0-MHz and 1.5-MHz signals transmitted simultaneously from a single source. Reference 11 reported detection of both the sum and difference frequencies as amplitude modulation of a standing mixed-frequency wave. The experiment was conducted in oil. A photograph of the signal pattern is included in the reference.

Bellin and Beyer used 7.4-MHz and 6.0-MHz sources in water and searched for the sum frequency. Various beam intersection angles were employed. A signal was detected only when the pickup was on the axis of one of the signals with beams perpendicular. The authors decided that the signal was psuedo-sound because of its low level. A scatterer was placed in the interaction region, and the difference frequency was detected at several locations outside of the region. It was supposed that this latter signal resulted from radiation pressures on the scatterer rather than from non-linear interaction in the medium. The results of this experiment, reported in Ref. 12, were thus contrary to those of Ingard and Pridmore-Brown. Bellin and Beyer attributed those results to psuedo-sound.

In Ref. 13, Bellin and Beyer reported an experiment with two colinear sound beams in a water-filled tank about 2 x 3 x 1 ft. A small quartz crystal was driven simultaneously at about 13 MHz and 14 MHz. 40 W

sources were used, producing an acoustic pressure of about 3 atm. The pickup was a 1/16-in. diameter barium titanate probe coupled to a communications receiver. The difference-frequency signal was detected on the axis. The beam width was very narrow. The signal strength was about 20,000 μbar. The authors recognized the possibility of psuedo-sound but argued analytically that its strength would not be sufficient to produce the signal recorded. The possibility of non-linearity in the circuitry was also rejected. Bellin and Beyer considered their results to be experimental verification of Westervelt's theory of an "end-fire array" [Ref. 7].

Berktay and Smith performed a similar experiment with a 9-cm^2 transducer operating at about 3 MHz. Their findings, reported in Ref. 14, were in close agreement with Westervelt's theory.

Dunn, Kuljis and Welsby conducted experiments with a pair of spherically-curved transducers. These were aligned with centers of curvature coincident. Up to 100 W were supplied to each unit in order to produce cavitation in the medium. Primary frequencies of 373 kHz and 326 kHz were employed. The experiments were conducted in a tank of about one cubic meter. In Ref. 15, the authors reported that a difference-frequency signal was produced and that its strength was enhanced by cavitation.

Sachs used high-energy beams at 1010 kHz and 1410 kHz and investigated the pressure field for various beam angles including colinearity. He found no evidence of interaction. The results were reported in Ref. 16.

Al-Temimi used 9-cm^2 transducers at 3.15 MHz and 2.85 MHz and aligned the primary beams to intersect at 20 degrees. The center of the

interaction region was 30-60 cm from the transducers. Primary acoustic powers of 15 W and 30 W were generated. The difference frequency was detected only along the primary beams. The results were reported in Ref. 17.

A few other experiments have been reported. Mostly, they have concerned parametric amplification or cylindrically/spherically spreading waves. The results which have been summarized here demonstrate the difficulties introduced by psuedo-sound. In one case, at least, even though the authors (Ingard and Pridmore-Brown) took account of psuedo-sound in their arrangement, there was an apparent misinterpretation of the data. Westervelt [Ref. 7] has stated that no signal will be scattered outside of the interaction region in the case of perpendicular intersection. Nevertheless, there remain reports of measurable signals in this case. In the work of Dunn, Kuljis and Welsby, one might suppose that cavitation nuclei served in the same manner as a hydrophone diaphragm to generate the difference frequency; alternatively, these nuclei may have served as scatterers of the primary signals, which in turn interacted in a non-linear fashion at the detector.

C. USE OF ADJACENT TRANSDUCERS

To provide further studies in the case of waves which are propagating in the same direction, transducers were chosen that were resonant at about 400 kHz. They were mounted side-by-side (detailed description below), supposing that the primary fields would overlap sufficiently to

approximate colinear, planar waves. That arrangement was also thought to be more adaptable to practical use in sonar arrays than a system requiring dual-frequency operation of individual elements. Sources operating at a few hundred kilohertz were chosen because they provided the best compromise between small transducers on the one hand and close control of the difference frequency on the other. A difference frequency of 50 kHz seemed to be in the proper range for possible practical applications.

D. EQUIPMENT DESCRIPTION

1. Transducers

The transducers were piezoelectric elements and were mounted inside a cylindrical brass casing filled with castor oil. The active diaphragm was rubber and was 2.5 cm in diameter. The units were mounted in rubber-insulated clamps and were suspended as shown in Fig. 4. The distance between diaphragm centers was 8 cm. The beams approximated those of a piston source (Fig. 5), and the near field was under one meter. Maximum electrical power used was 0.153 W. The transducers were operated at 385 kHz and 435 kHz. The units were reversible.

2. Hydrophone

The hydrophone used to search for a difference-frequency signal was a reversible rectangular barium-titanate unit with an active area of 5.5 cm x 1 cm. The hydrophone body was coated with neoprene. See

Fig. 6. The hydrophone was sharply resonant at 50 kHz with a response (determined by reciprocity calibration) of 1.63×10^{-5} V/μbar. Its response at the primary frequencies was less by a factor of ten.

3. Anechoic Tank

The experiments were conducted in a fresh-water tank of length 7.5 m, width 1.9 m and water depth 2.0 m. The tank sides and bottom were lined with rubber/wood sound-absorbing material. A view of the tank is given in Fig. 7.

Since the transducers were mounted only about 24 cm below the water surface, reflection effects were anticipated. Figure 8 shows the relative values of pressure along the length of the tank, measured at the primary frequencies and at constant depth. The undulating interference pattern is apparent. To minimize the effect of this variation during data collection, the hydrophone was always placed at the depth which gave a maximum difference-frequency signal.

The transducers were fixed near one end of the tank with the axis of the dual beam 32 cm from the tank side (Fig. 9). The hydrophone was moved to various points along the axis. It was mounted as shown in Fig. 10.

4. Circuitry

See Fig. 11.

E. THE SIGNAL

Displayed on a cathode ray oscilloscope (CRO), the hydrophone output was similar to an amplitude-and phase-modulated carrier. As

indicated by the photographs of Figs. 12 and 13, the amplitude modulation percentage dropped if the amplitude of a primary signal was reduced. In these figures, and in Figs. 14 and 15, it can be seen that the phase modulation was also reduced if a primary signal strength was reduced. For full power to the primary transducers, the maximum "carrier" frequency indicated was about 410 kHz, and the minimum was about 350 kHz. The "envelope" frequency was 50 kHz, corresponding to the difference between the two primary frequencies. A graphical Fourier analysis showed that the "envelope" waveform included first, second and third harmonics. Figures 16 and 17 show the distortion of the total waveform which could be produced by slight adjustments of the hydrophone position about the axis. No direct correlation could be established between the amount of this distortion and the strength of the difference-frequency signal.

A wave analyzer was used as a filtered network to search for the difference-frequency signal. The hydrophone was placed at various distances along the axis of the primary beams. The strength of the primary signals there was measured separately. When tuned to the difference frequency, the wave analyzer meter displayed periods of erratic fluctuation separated by short intervals of comparatively steady voltage level. The wave analyzer output, viewed on the CRO, was a 50-kHz sinusoid which waxed and waned similarly. The maximum steady rms voltage was recorded. The results are plotted in Fig. 18. The amplitude of the difference-frequency signal varied essentially as the strength of

the primary signals. With increasing distance past the four-meter point, the difference-frequency signal dropped a little faster than the primary signal.

Measurements of the difference-frequency signal and the primary signals were also made in azimuth. Figure 19 shows that the difference-frequency signal beam was somewhat narrower than the primary beam.

It was noted that the noise level in the wave analyzer was generally higher when the primary sources were energized than when they were deenergized. It is supposed that subharmonics were present, but none were detected.

F. COMPARISON

To compare experimental data with the theory of interaction of plane waves, it was necessary to assume bounds for the interaction volume cross-section. The square area defined by 12-degree arcs at one meter was chosen, as it would include only points with pressures within 3 dB of the axial pressure. An area of about $0.04\ m^2$ was thus defined. The axial pressure at one meter was measured for each primary signal. A separate calibrated hydrophone was employed. Pressures of 2990 μbar and 2470 μbar were recorded, corresponding to acoustic intensities of $0.061\ W/m^2$ and $0.041\ W/m^2$. For the area defined above, the values for W_1 and W_2 in Eq. (20) became 2.44 mW and 1.64 mW.

For $f_1 = 435$ kHz and $f_2 = 385$ kHz, $f_1 - f_2 = 50$ kHz; $A \approx \alpha_1 + \alpha_2 - \alpha = 1 \times 10^{-2}$ nepers/m; $\alpha = 10^{-4}$ nepers/m; $R = 6$ m; $c_o = 1480$ m/sec.

Proceeding,

$$P_s = \frac{\sqrt{W_1 W_2}\,(2\pi)(f_1 - f_2)^2\, e^{-\alpha R}}{c_o^3\, RA} = 1.6\ \mu\text{bar}$$

With a factor of three for the expression in brackets in Eq. (12), this value would be increased to 4.8 μbar.

From the voltage level noted in Fig. 18 and from the calibration data, the experimentally determined pressure was found to be $P = 21.6\ \mu$bar.

V. ANALYSIS OF RESULTS

The linear addition of two acoustic waves of different frequencies produces a complex harmonic signal. Reference 18 shows that when two signals of nearly the same frequency are added, the resultant average frequency is somewhere between the two component frequencies. The phase angle of the signal will vary with time, depending on the relative amplitudes of the component signals, so that the oscillation is not sinusoidal. The amplitude of the resultant will vary between the sum and the difference of the component amplitudes, at a rate equal to the difference of the two frequencies.

If the two signals are summed by a linear device and the resultant is fed to a spectrum analyzer, the latter device will show the amplitudes of the two components. If the two signals are summed by a non-linear device, the spectrum analyzer will then show components at the original frequencies, at the sum frequency, at the difference frequency, etc.

The signal of this experiment, as shown in Figs. 12-17, appears to be that waveform described in the first paragraph. The amplitude and phase variations are evident. The wave analyzer showed, however, that a component at the difference frequency was present at the hydrophone output. Non-linear interaction in the medium and/or in the hydrophone was the probable origin of that component.

To evaluate the linearity of the hydrophone, a separate test was arranged. First, with one of the primary transducers operating at 385 kHz

and with the hydrophone at a distance of about two meters in the tank, hydrophone output was recorded for various transducer input voltages. The response is plotted in Fig. 20. For independent measurements of the primary field at two meters, a third reversible transducer was employed as a receiving hydrophone. Its output is plotted. A similar test was conducted at 435 kHz. All of the lines in Fig. 20 are seen to diverge. If the primary field lines are made parallel and hydrophone response lines are shifted the same amount as the corresponding field lines, the hydrophone response lines are seen to diverge, still. The hydrophone response to incremental pressure changes was thus different for the two frequencies. This would indicate that the hydrophone was not precisely linear.

The slight non-linearity in the hydrophone response suggests that the difference-frequency signal originated in part in the hydrophone. Without destructive disassembly and testing of the isolated diaphragm, the nature of its summing process cannot be stated. The magnitude of the psuedo-sound contribution thus remains undetermined.

To further explore the production of a difference-frequency signal in a hydrophone, the third reversible transducer was placed in the combined primary fields. The output of that unit, when studied with the wave analyzer, showed a difference-frequency component. Its amplitude was about 1/20th of that recorded with the regular hydrophone, although the reversible transducer showed no measurable output when subjected to a 50-kHz signal alone.

Finally, it should be noted that the "modulated carrier" waveform was also produced when the primary transducers were oriented at various angles. The hydrophone was placed at various positions in and out of the interaction region, and the amplitude of the waveform varied with hydrophone proximity to the primary beams.

The computation of the theoretical amplitude of the difference-frequency signal is not precise. Equations (16) and (17) were developed for colinear plane waves, whereas adjacent piston sources were used in the experiment. Primary pressures were taken as those existing at one meter from the source, whereas actual pressures were much greater at near-field peaks and less at greater distances because of spreading. As explained in section III., the effective interaction region is shortened if piston sources are used. The effect must be to reduce the amplitude of the difference-frequency signal below that which is computed from Eq. (20).

Although a 50-kHz signal was recorded, the evidence suggests that generation in the hydrophone may have been of greater significance than generation in the medium. The measured amplitude was greater than the predicted value which was computed for the ideal case. The amplitude was still so small that the signal would not be discernible when superimposed on the "modulated carrier" waveform.

VI. OTHER EXPERIMENTAL TECHNIQUES

In the early part of the experimental work it became apparent that available hydrophones which were sensitive in the 10-50 kHz range were also sensitive at the primary frequencies. The result was the "modulated carrier" waveform. Attempts were made to eliminate the primary signals at the hydrophone while preserving any difference-frequency signal which might have existed.

A. ACOUSTICAL FILTERS

When the "modulated carrier" was first detected, a steel plate was mounted in the tank to reflect the signal 90 degrees. A grid was constructed with layers of screen wire of about 1/4-in. mesh. This was then mounted on the plate. Cavities of about 1/4-in. depth were thus introduced on the surface of the reflector. It was supposed that the primary signals might be attenuated significantly, while the longer wavelength difference-frequency signal might be reflected with little attenuation. The primary signals were in fact attenuated noticeably, however the nature of the waveform was unchanged.

Returning to the direct path, various thicknesses of steel plate were mounted between the sources and the hydrophone. The object was to find a plate which would reflect the primary signals while passing the lower-frequency signal. A 3/32-in. plate was found to reduce the higher-frequency signal by about 90%, but to pass a signal at the difference frequency with only about 10% reduction. When only the primary

signals were directed onto the plate, however, the usual waveform (reduced in amplitude) was again displayed. No additional signal was evident.

B. PULSE TECHNIQUES

The signal generators were set for gated operation, and a pulse of about 100 μsec was produced. A reflecting path totaling three tank lengths was arranged, and the CRO was synchronized with the signal sources. By reducing the amplitude of the primary signals at the source, and then allowing attenuation with distance, it was thought that the primary signal might be "screened" out. The lower-frequency signal, subject to less attenuation, might still be detectable after the multiple reflections. Only the high-frequency pulse, reduced in amplitude after each reflection, was observed, however.

C. VARIATION WITH DIFFERENCE FREQUENCY

Berktay and Smith, in Ref. 19, reported a study of cylindrically spreading waves. They discriminated between psuedo-sound and the desired signal by noting that if the difference frequency was increased, their signal amplitude increased. Since they were tuning the primary sources off of resonance, the primary signals (and psuedo-sound) would have been reduced.

VII. CONCLUSIONS

The existence of a true acoustic signal at the difference frequency was not shown conclusively in the experimental work. There was evidence of non-linearity in the hydrophone, and the measured signal may have included signals generated in the medium and in the hydrophone. The theory of plane-wave interaction was neither confirmed nor disproved.

Further studies in the kilohertz range are warranted, but more refined experimental techniques should be used. Typical hydrophones will be sensitive to the primary frequencies as well as to the difference frequency, and smaller probes should be used. The center of the interaction volume should be studied without subjecting the receiving unit to full primary pressures over an area of several square wavelengths. Higher powers than those obtainable with the WAVETEK units should be applied. This might raise the level of the desired signal so that it could be distinguished from psuedo-sound. Also, the transducers and hydrophone should be located at a greater depth to eliminate the perturbations of surface reflections.

The interaction of two sound beams remains, in theory, as potentially useful in sonar applications. The fact of low efficiency limits the general application of the process. Efficiency is a sensitive function of frequency and of primary transducer design. The "modulated carrier" waveform may be useful in its own right as, for example, in an unique signal-processing scheme.

APPENDIX A: FIGURES

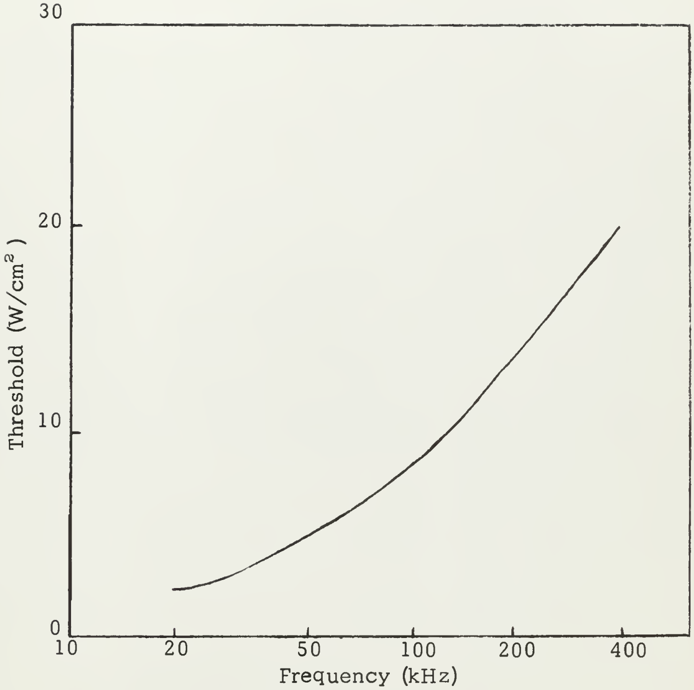


Figure 1. Cavitation threshold in water as a function of frequency.

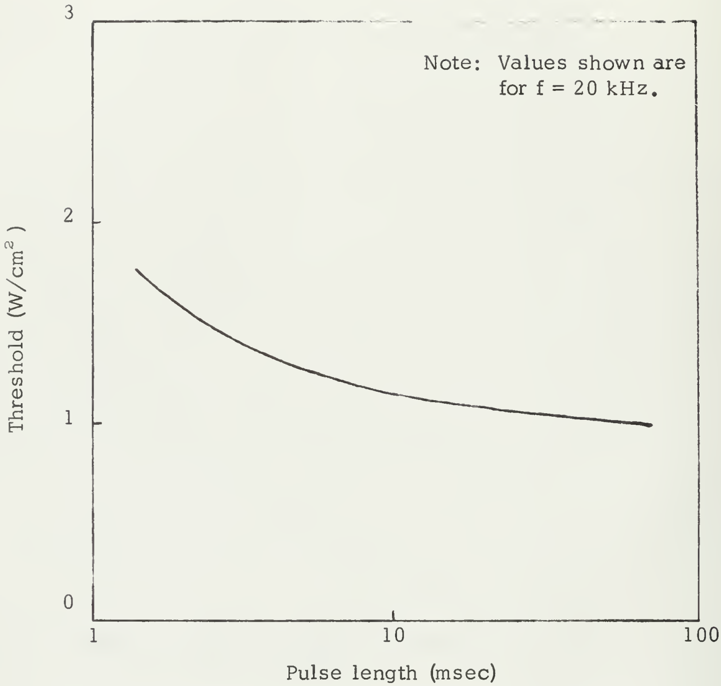


Figure 2. Cavitation threshold in water as a function of pulse length.

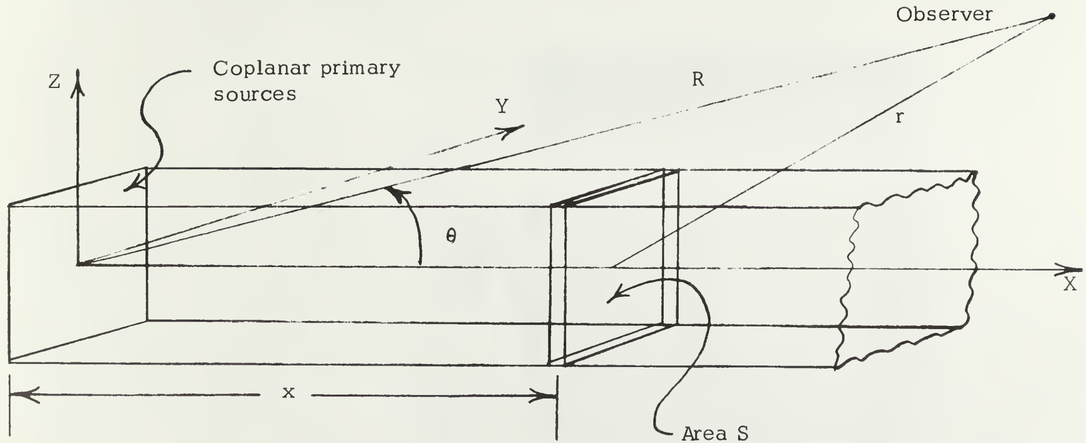


Figure 3. Geometry of interacting plane waves.

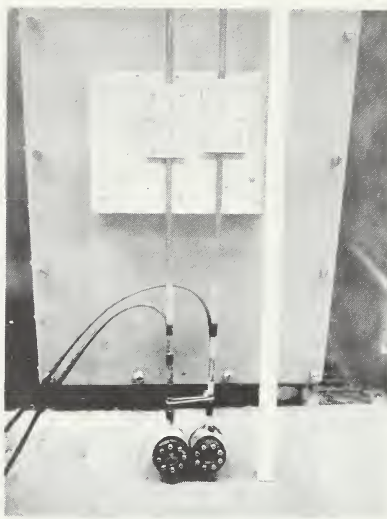


Figure 4. Transducer mounting

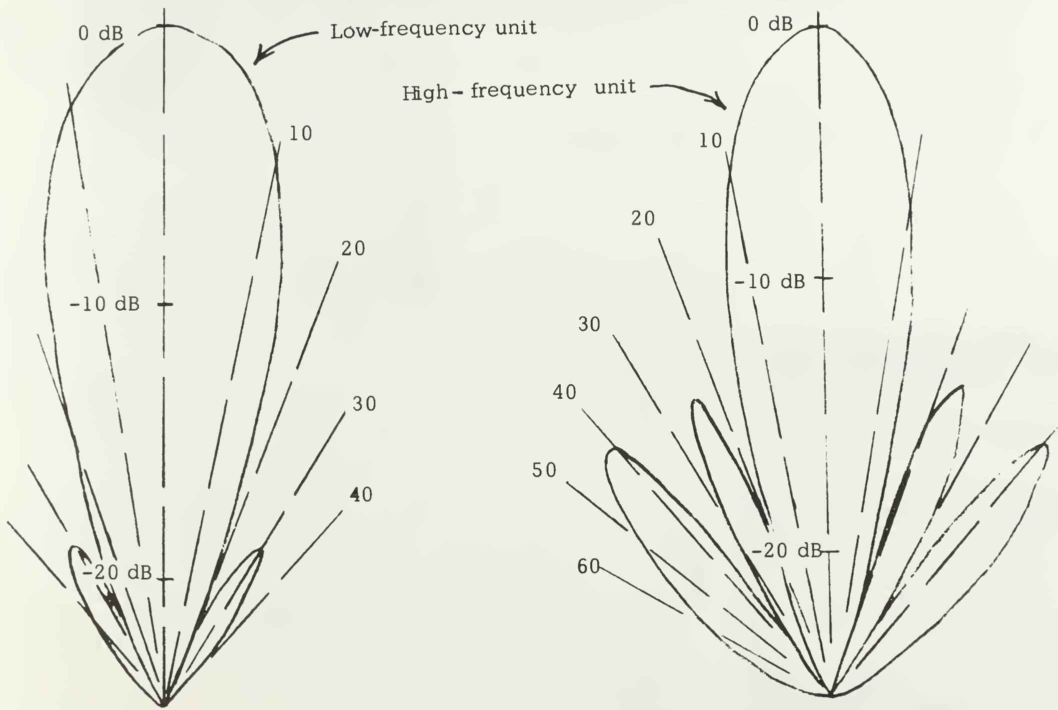


Figure 5. Smoothed beam patterns of primary sources (symmetry assumed).

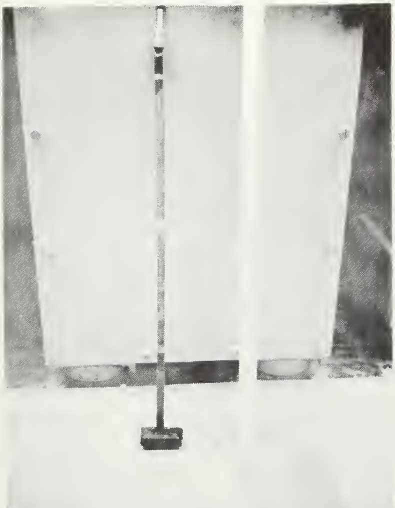


Figure 6. Hydrophone.

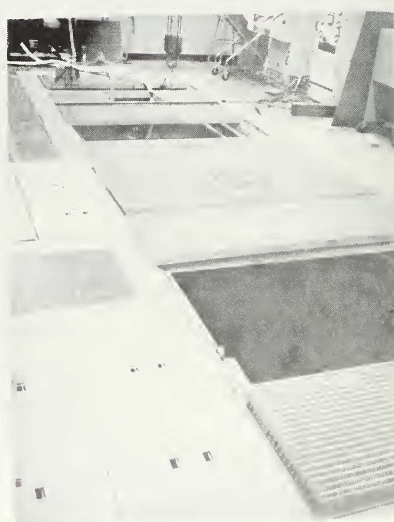


Figure 7. Anechoic tank.

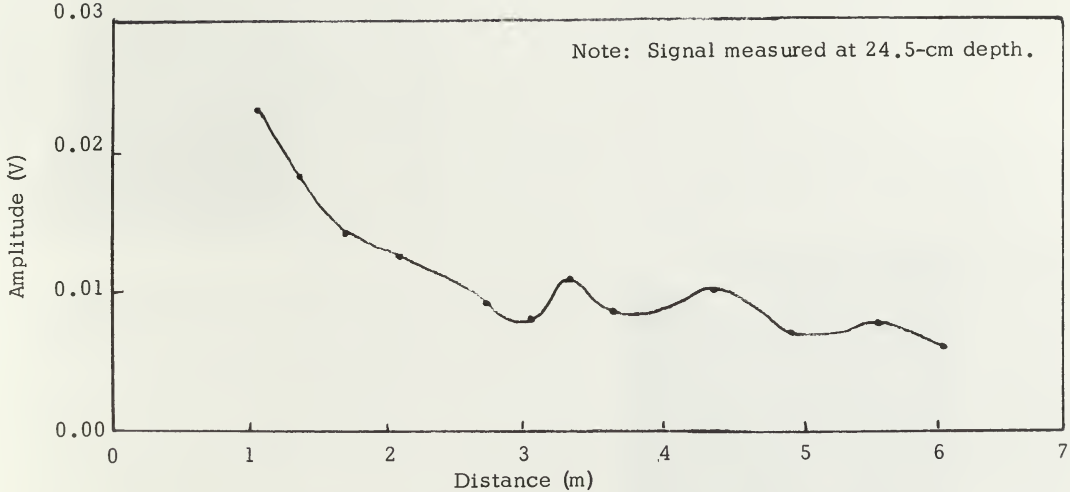


Figure 8. Primary signal-strength variation as a result of surface reflection effects.

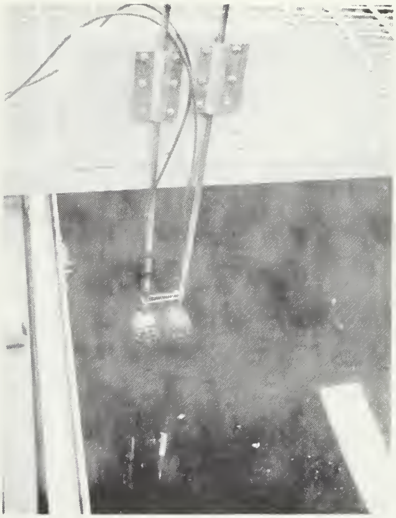
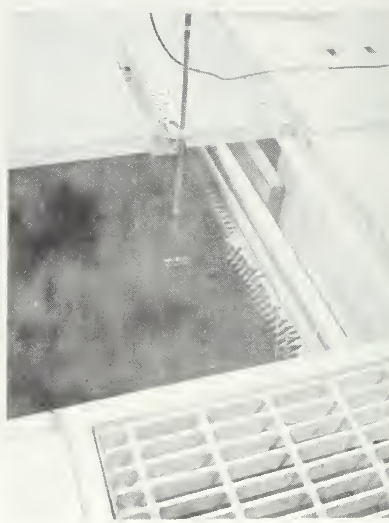


Figure 9. Transducer mounting.

Figure 10. Hydrophone mounting.



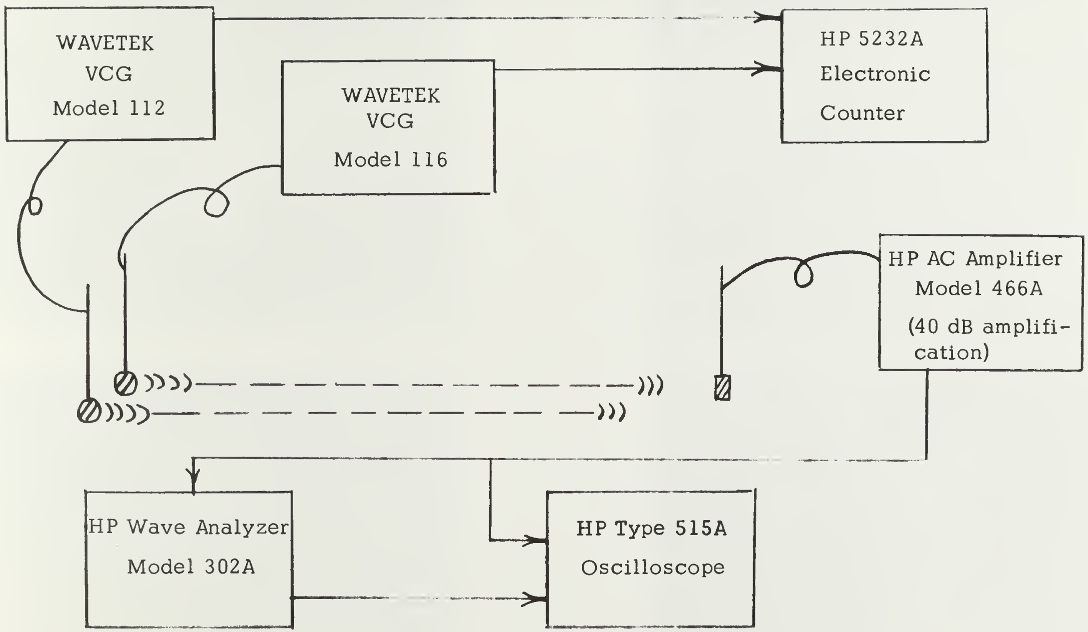


Figure 11. Equipment arrangement.

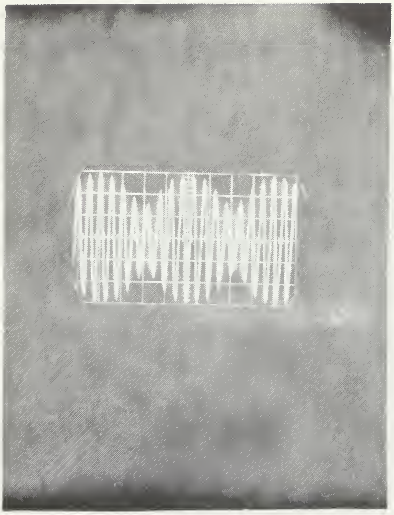


Figure [illegible]. Waveform of signal with each transducer at full power.

Scale:

[illegible]

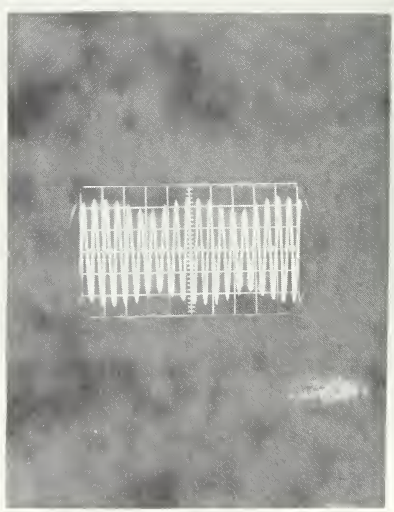


Figure [illegible]. Waveform of signal with one transducer at full power and the other at 1/2 power.

Scale:

[illegible]

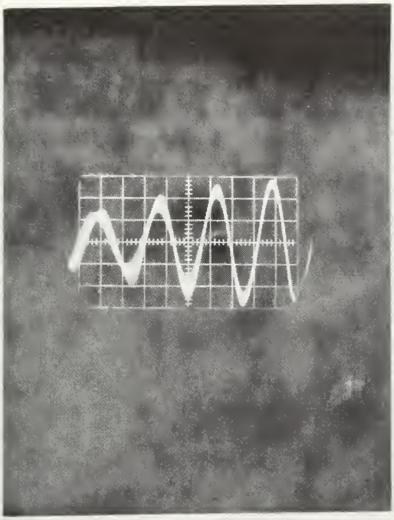


Figure 14. Waveform of signal (magnified) with both transducers at full power.

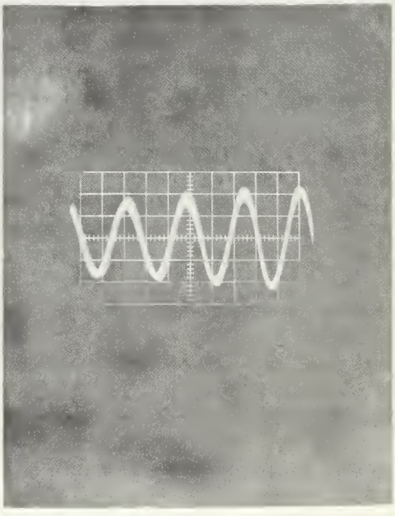
Scales:

1 μsec/cm
0.1 V/cm

Figure 15. Waveform of signal (magnified) with one transducer at full power and the other at half power.

Scales:

1 μsec/cm
0.1 V/cm



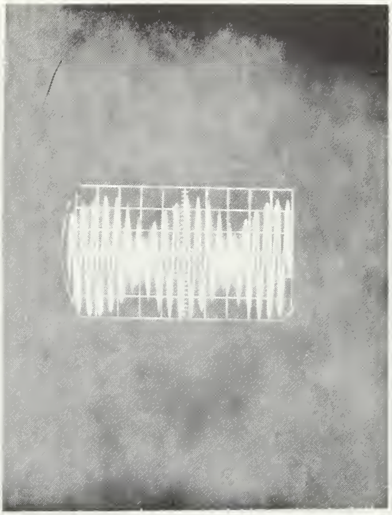


Figure 16. Distortion in waveform

Scales:

5 μsec/cm
0.1 V/cm

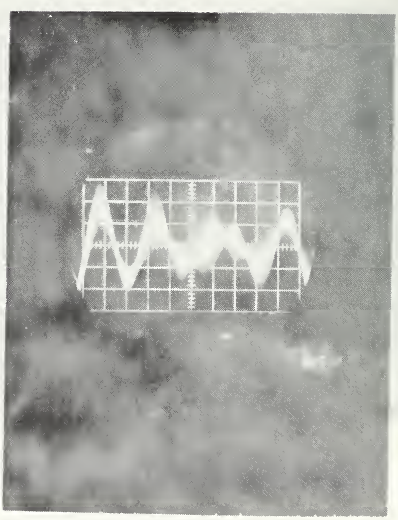


Figure 17. Distortion in waveform (display magnified)

Scales:

1 μsec/cm
0.1 V/cm

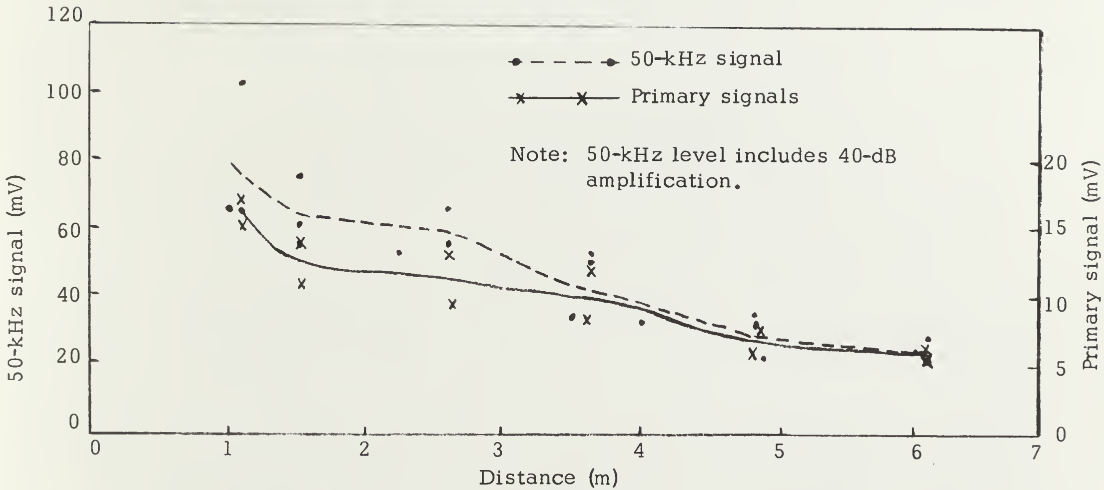


Figure 18. Primary signal and difference-frequency signal variation with distance.

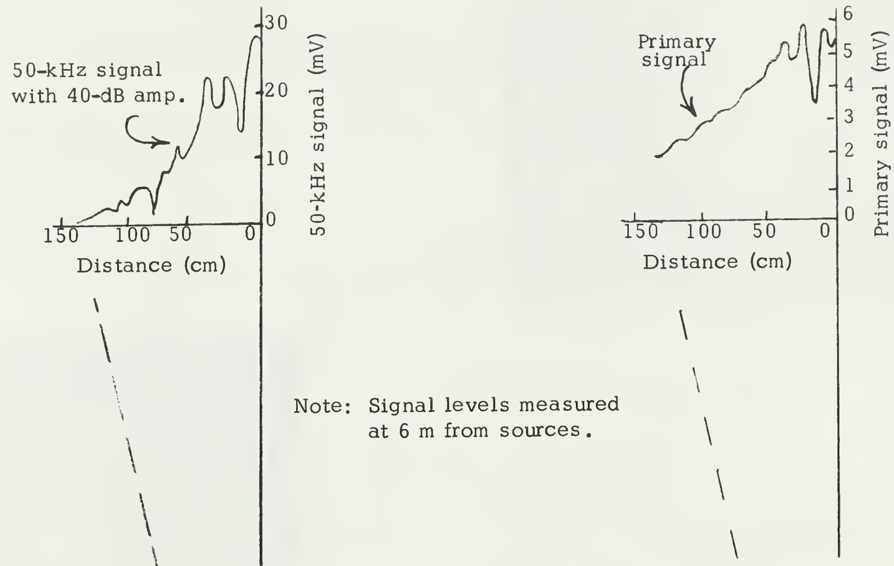


Figure 19. Primary and difference-frequency signals vs. distance from axis.

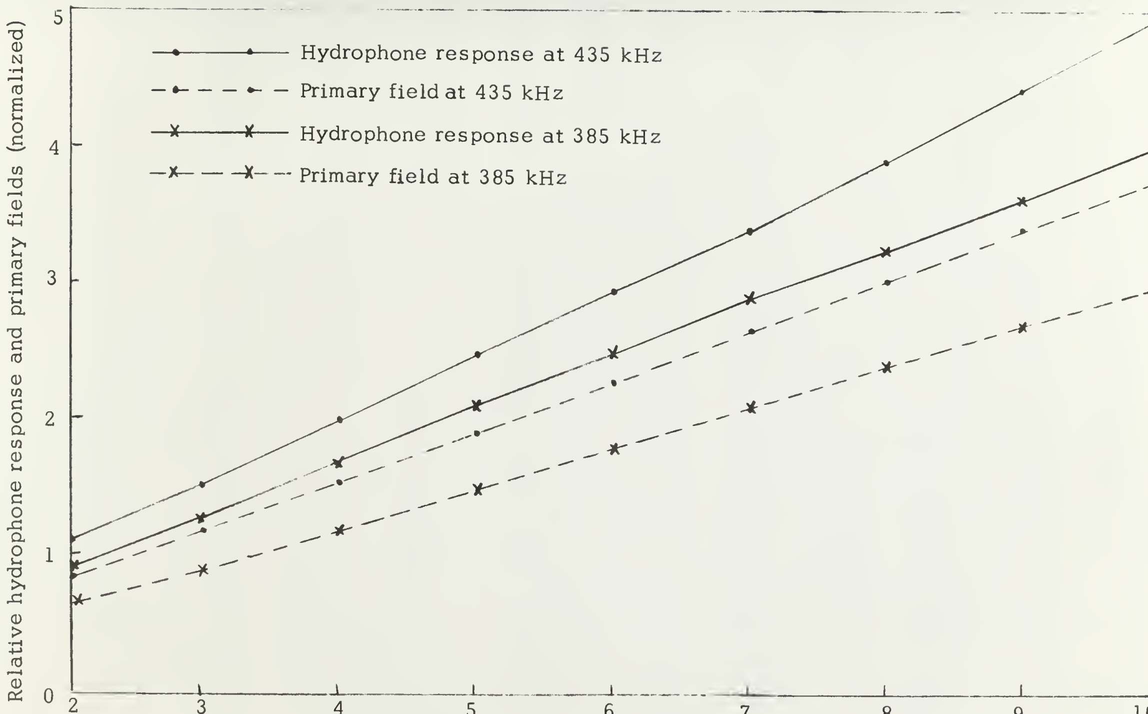


Figure 20. Hydrophone response to individual primary signals; primary fields included for reference.

LIST OF REFERENCES

1. Tucker, D. G., "The Exploitation of Non-Linearity in Underwater Acoustics," J. Sound Vib., v.2, p.429-434, 13 April 1965.

2. Berktay, H. O., "Possible Exploitation of Non-Linear Acoustics in Underwater Transmitting Applications," J. Sound Vib., v.2, p.435-461, 13 April 1965.

3. Berktay, H. O., "Parametric Amplification by the Use of Acoustic Non-Linearities and Some Possible Applications," J. Sound Vib., v.2, p.462-470, 13 April 1965.

4. Berktay, H. O., "Some Proposals for Underwater Transmitting Applications of Non-Linear Acoustics," J. Sound Vib., v.6, p.244-254, 8 November 1966.

5. Lighthill, M. J., "On Sound Generated Aerodynamically," Proc. R. Soc., v. A211, p.564-587, 13 November 1951.

6. Westervelt, P. J., "Scattering of Sound by Sound," J. Acoust. Soc. Am., v.29, p.199-203, 19 October 1956.

7. Westervelt, P. J., "Parametric Acoustic Array," J. Acoust. Soc. Am., v.35, p.535-537, 28 December 1962.

8. Lauvstad, V. and Tjötta, S., "Nonlinear Interaction of Two Soundbeams," J. Acoust. Soc. Am., v.35, p.929-930 (L), 18 February 1963.

9. Gorelik, A. G. and Zverev, V. A., "On the Problem of Mutual Interaction Between Soundwaves," Sov. Phys. Acoust., v.1, p.353-357, 4 May 1955.

10. Ingard, U. and Pridmore-Brown, D. C., "Scattering of Sound by Sound," J. Acoust. Soc. Am., v.28, p.367-369, 17 February 1956.

11. Mikhailov, G. D., "The Phenomenon of Amplitude Modulation in Mixed-Frequency Acoustic Waves," Sov. Phys. Acoust., v.3, p.406-409, 29 April 1957.

12. Bellin, J. L. S. and Beyer, R. T., "Scattering of Sound by Sound," J. Acoust. Soc. Am., v.32, p.339-341, 7 December 1959.

13. Bellin, J. L. S. and Beyer, R. T., "Experimental Investigation of an End-Fire Array," J. Acoust. Soc. Am., v.34, p.1051-1054, 9 April 1962.

14. Berktay, H. O. and Smith, B. V., "End-Fire Array of Virtual Sound Sources Arising from the Interaction of Sound Waves," Electronics Letters, v.1, p.6, 28 January 1965.

15. Dunn, D. J., Kuljis, M. and Welsby, V. G., "Non-Linear Effects in a Focused Underwater Standing Wave System," J. Sound Vib., v.2, p.471-476, 14 May 1965.

16. Sachs, T. D., "Mutual Interactions of High-Energy Ultrasonic Beams," J. Acoust. Soc. Am., v.33, p.857 (A).

17. Al-Temimi, C. A., "Interaction Between Two Sound Fields Propagating in Different Directions," J. Sound Vib., v.8, p.44-63, 14 November 1967.

18. Kinsler, L. E. and Frey, A. R., Fundamentals of Acoustics, 2nd ed., p.12-15, 402-403, Wiley, 1962.

19. Berktay, H. O. and Smith, B. V., "End-Fire Array of Virtual Sources Produced by the Interaction of Cylindrically Spreading Acoustic Waves," Electronics Letters, v.1, p.202, 13 August 1965.

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13. ABSTRACT

Modern sonars are large in size. This is a result of several factors connected with operation at low frequencies. Non-linear interaction of two high-frequency sound beams in water may provide a source of low-frequency sound without the use of large transducers. The theory of interaction of two plane waves is reviewed, and conversion efficiency is studied. Early experimental work is also reviewed. To provide new experimental data, primary sources of 385 kHz and 435 kHz are mounted side-by-side in an anechoic tank. The sound field between the parallel beams is explored. A difference-frequency signal is detected, but its amplitude is found to be greater than the theoretical value. Non-linearities in the receiving hydrophone are suspected as contributing to the 50-kHz signal level. Various techniques for eliminating psuedo-sound are discussed, and recommendations for further study are presented.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
sonar						
cavitation						
interaction of sound beams						
difference frequency signal						
psuedo-sound						